Analysis of an Elliptical Conducting Rod Between Parallel Ground Planes by Conformal Mapping

B. N. DAS AND K. V. SESHAGIRI RAO

Abstract—**The paper presents a conformal mapping analysis of an elliptic conducting rod between parallel ground planes, where one of the principal axes of the rod is parallel to but not necessarily centered between the ground planes. The conditions under which this analysis can be applied to the cases of planar and circular conductors between ground planes are obtained. Also, the formulation is extended to the special case of the conductor above a single ground plane.**

I. Introduction

TRANSMISSION-line geometries consisting of a planar strip conductor between parallel ground planes have been studied using both analytical and numerical methods [1]–[4]. Impedance data for transmission lines with circular and elliptic inner conductors symmetrically located between ground planes have also been reported in the literature [5]–[9]. Also, Wheeler has suggested a method of impedance evaluation for a number of other generalized structures [10].

In the present work, the conformal transformation of a conductor of elliptic cross section between the ground planes into a parallel-plate configuration is developed. One axis of the ellipse is oriented parallel to ground planes and is displaced from the plane of symmetry. The formulation is extended to the cases of asymmetrically located conductors of circular cross section and planar conductors along either of the principal axes of the ellipse. This analysis leads to a set of equations from which the characteristic impedances of all the above structures can be determined. The impedance data for two displaced positions are presented in the form of charts which can be used to obtain the characteristic impedance of all the above structures.

The general formulation is then used to obtain the conformal transformation for the case when one of the ground planes is moved to infinity. The set of equations obtained for general case reduce to a new set of equations from which an impedance chart is obtained. For the particular case of a conductor of circular cross section above a ground plane, the impedance data are compared with those calculated using the transformation suggested by Decreton [11].

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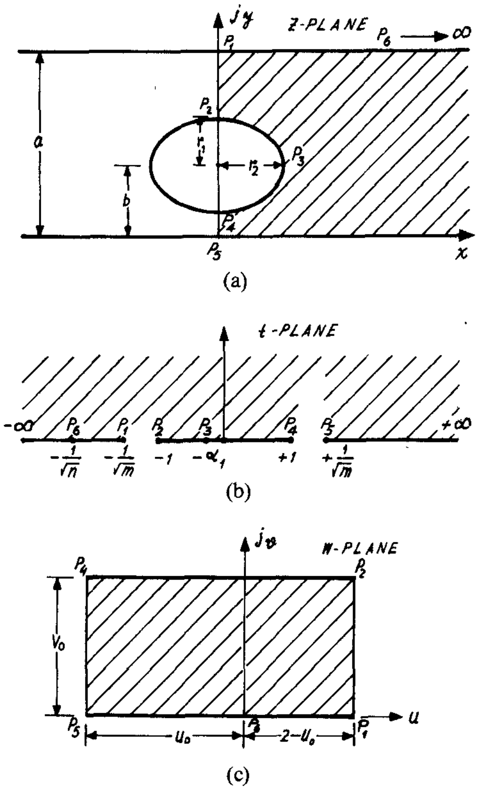

Fig. 1.

II. Conformal Transformation

Fig. 1(a) shows the coordinate system of a structure which consists of a conductor with a curved boundary located between the ground planes. The boundary of the conductor is assumed to have structural symmetry with respect to its principal axes which are oriented parallel to y and x axes. The principal axis parallel to the x-axis is displaced with respect to the plane of symmetry between the ground planes. A Schwarz–Christeffd transformation which transforms the upper half of Fig. 1(b) into the shaded portion of Fig. 1(a) is given by

$$Z = C_0 \int_0^t \frac{\left(t + \frac{(1-A_1)}{\sqrt{n}}\right) - \lambda \cdot \sqrt{t^2-1}}{\left(t + \frac{1}{\sqrt{n}}\right)\sqrt{(t^2-1)\left(t^2 - \frac{1}{m}\right)}} + B_1 \qquad (1)$$

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where A_1, B_1, C_0, λ, m, and n are constants

$$0<m<1 \qquad m>n \quad \text{and} \quad 0 \leqslant \frac{(1-A_1)}{n}<1.$$

The term

$$\left[t+\frac{(1-A_1)}{\sqrt{n}}-\lambda\cdot\sqrt{t^2-1}\right]$$

is the curve factor [12].

Carrying on integration, (1) takes the form

$$Z=x+jy=C_1\Big[u-A_1\big\{\pi(n,u|m)-\sqrt{n}\,f(m,n,u)\big\} + j\sqrt{n}\,\lambda g(m,n,u)\Big]+B_1 \qquad (2)$$

where $C_1=C_0\sqrt{m}$, $t=\sin\Phi=\operatorname{sn}u$, $u=F(\Phi|m)$ is the incomplete elliptic integral of the first kind and $\pi(n;u|m)$ is the incomplete elliptic integral of the third kind. The expressions for $f(m,n,u)$ and $g(m,n,u)$ are given by

$$f(m,n,u)=\frac{1}{2\sqrt{(1-n)(m-n)}}\ln\left[\frac{2(1-n)(m-n)+(1-n\operatorname{sn}^2u)(n+mn-2m)+2n\sqrt{(1-n)(m-n)}\operatorname{cn}u\operatorname{dn}u}{n\left(2n-m-1+2\sqrt{(1-n)(m-n)}\right)(1-n\operatorname{sn}^2u)}\right] \qquad (3)$$

$$g(m,n,u)=\frac{1}{\sqrt{m-n}}\left[\sin^{-2}\frac{\sqrt{n}+m\operatorname{sn}u}{\sqrt{m}\left(1+\sqrt{n}\operatorname{sn}u\right)}-\sin^{-1}\frac{\sqrt{n}}{\sqrt{m}}\right] \qquad (4)$$

where sn$u$, cn$u$, and dn$u$ are elliptic functions.

Boundary conditions required for the evaluation of the constants in (2) are obtained from the coordinates of

$$P_1\left(Z=ja, t=-\frac{1}{\sqrt{m}}\right)$$
$$P_2(Z=j(b+r_1), t=-1)$$
$$P_3(Z=(r_2+jb), t=-\alpha_1)$$
$$P_4(Z=j(b-r_1), t=+1)$$
$$P_5\left(Z=0, t=+\frac{1}{\sqrt{m}}\right).$$

Substitution of the boundary conditions at the points shown in Fig. 1(a) into (2) and solving the resulting set of equations results in the following relations among the constants $A_1, B_1, C_1, \lambda, m, n$, the ground plane spacing a, the separation between strip and lower ground plane b, and the semimajor and minor axes r_1 and r_2 of the ellipse

$$A_1=\frac{K(m)}{\pi(n,K(m)|m)} \qquad (5a)$$

$$G_1=\frac{-a\sqrt{(m-n)(1-n)}}{\sqrt{n}\,\pi\left(A_1+\sqrt{1-n}\cdot\lambda\right)} \qquad (5b)$$

$$B_1=\frac{aA_1\ln(M_1)}{2\pi\left(A_1+\sqrt{1-n}\cdot\lambda\right)}+j\left[b+\frac{a\sqrt{1-n}\cdot\lambda}{2\pi\left(A_1+\sqrt{1-n}\cdot\lambda\right)}\cdot\left\{\sin^{-1}\frac{-m+\sqrt{n}}{\sqrt{m}\left(1-\sqrt{n}\right)}+\sin^{-1}\frac{m+\sqrt{n}}{\sqrt{m}\left(1+\sqrt{n}\right)}-2\sin^{-1}\frac{\sqrt{n}}{\sqrt{m}}\right\}\right] \qquad (5c)$$

$$\frac{2b}{a}=1-\frac{A_1F\left(\sin^{-1}\sqrt{\frac{n}{m}}\,\Big|\,m\right)}{\left(A_1+\sqrt{1-n}\cdot\lambda\right)K(m)}+\frac{\sqrt{1-n}\cdot\lambda}{\pi\left(A_1+\sqrt{1-n}\cdot\lambda\right)}\cdot\left\{\sin^{-1}\frac{-m+\sqrt{n}}{\sqrt{m}\left(1-\sqrt{n}\right)}+\sin^{-1}\frac{m+\sqrt{n}}{\sqrt{m}\left(1+\sqrt{n}\right)}\right\} \qquad (5d)$$

$$\frac{r_1}{a}=\frac{\sqrt{1-n}\cdot\lambda}{2\pi\left(A_1+\sqrt{1-n}\cdot\lambda\right)}\cdot S(m,n) \qquad (5e)$$

$$\frac{r_2}{a}=\frac{\sqrt{(m-n)(1-n)}}{\sqrt{n}\,\pi\left(A_1+\sqrt{1-n}\cdot\lambda\right)}\cdot Q(m,n) \qquad (5f)$$

and

$$\lambda=\frac{2\sqrt{m-n}\cdot Q(m,n)}{\sqrt{n}\cdot S(m,n)}\cdot\frac{r_1}{r_2} \qquad (5g)$$

where $K(m)$ and $\pi(n;K(m)|m)$ are complete elliptic integrals of the first and third kind, respectively, $\pi=3.14159$, and

$$S(m,n)=\sin^{-1}\frac{m+\sqrt{n}}{\sqrt{m}\left(1+\sqrt{n}\right)}-\sin^{-1}\frac{-m+\sqrt{n}}{\sqrt{m}\left(1-\sqrt{n}\right)} \qquad (5h)$$

$$Q(m,n)=F\left(\sin^{-1}\alpha_1|m\right)-A_1\pi\left(n;\sin^{-1}\alpha_1|m\right)+\frac{\sqrt{n}\cdot A_1\ln M_1}{2\sqrt{(m-n)(1-n)}}-\sqrt{n}\,A_1M_2 \qquad (5i)$$

$$M_1=\frac{(1-m)}{\left(1+m-2n-2\sqrt{(m-n)(1-n)}\right)} \qquad (5j)$$

$$M_2=\frac{1}{2\sqrt{(1-n)(m-n)}}\ln\left[\frac{2(1-n)(m-n)+(n+mn-2m)(1-n\alpha_1^2)+2n\sqrt{(1-n)(m-n)}\sqrt{1-\alpha_1^2}\cdot\sqrt{1-m\alpha_1^2}}{n\left(2n-m-1+2\sqrt{(m-n)(1-n)}\right)(1-n\alpha_1^2)}\right] \quad (5k)$$

$$\alpha_1=\frac{\sqrt{n}-\sqrt{m}\cdot\Delta}{m-\sqrt{mn}\cdot\Delta} \quad (5l)$$

and

$$\Delta=\sin\left[\frac{1}{2}\left\{\sin^{-1}\frac{-m+\sqrt{n}}{\sqrt{m}\left(1-\sqrt{n}\right)}+\sin^{-1}\frac{m+\sqrt{n}}{\sqrt{m}\left(1+\sqrt{n}\right)}\right\}\right]. \quad (5m)$$

The curved boundary is transformed to a portion of the real axis of the t plane for which $|t|\leqslant 1$. For $|t|\leqslant 1$, u, $\pi(n;u|m)$, $f(m,n,u)$, and $g(m,n,u)$ are real. (2) can, therefore, be easily separated into real and imaginary parts which are given by

$$R=C_1\left[u-A_1\left\{\pi(n;u|m)-\sqrt{n}f(m,n,u)\right\}\right]+\frac{aA_1\cdot\ln(M_1)}{2\pi\left(A_1+\sqrt{1-n}\cdot\lambda\right)} \quad (6a)$$

$$I=b+C_1\sqrt{n}\lambda g(m,n,u)+\frac{a\cdot\lambda\sqrt{1-n}}{2\pi\left(A_1+\sqrt{(1-n)}\cdot\lambda\right)}\cdot\left\{\sin^{-1}\frac{-m+\sqrt{n}}{\sqrt{m}\left(1-\sqrt{n}\right)}+\sin^{-1}\frac{m+\sqrt{n}}{\sqrt{m}\left(1+\sqrt{n}\right)}-2\sin^{-1}\frac{\sqrt{n}}{\sqrt{m}}\right\}. \quad (6b)$$

From (2), (5e), (5f), (6a), and (6b) the coordinates x and y of a point on the curved boundary which are respectively the real and imaginary parts of the complex variable Z satisfy the equation of the form

$$\left(\frac{x}{r_2}\right)^2+\left(\frac{y-b}{r_1}\right)^2=A^2(m,n,\lambda) \quad (7)$$

where the expression A^2 is given by

$$A^2=\left(\frac{R}{r_2}\right)^2+\left(\frac{I-b}{r_1}\right)^2.$$

The right-hand side of (7) is numerically evaluated for t ranging from -1 to $+1$, $0<m\leqslant 0.99, 0\leqslant\lambda<\infty$, and the values of b/a equal to 0.1,0.4. Computed results reveal that depending upon the value of b/a, $A(m,n,\lambda)$ is very close to unity for certain range of values of m. The boundary of the conductor assumes the form of an ellipse under this condition. But this cross section near one or two ground planes departs from the usual ellipse. For a particular value of m and b/a, the lengths of principal axes of the ellipse depend on λ. Depending upon the orientation of the principal axes, the eccentricities of the ellipse are given by

$$e_1=\sqrt{1-\left(\frac{r_1}{r_2}\right)^2}, \quad \text{for } r_2\geqslant r_1 \quad (8a)$$

$$e_2=\sqrt{1-\left(\frac{r_2}{r_1}\right)^2}, \quad \text{for } r_2\leqslant r_1. \quad (8b)$$

The transformation from the upper half plane of Fig. 1(b) to the structure in Fig. 1(c) is given by [13]

$$W=u+jv=K_1\int_0^t\frac{dt}{\sqrt{(1-t^2)(1-mt^2)}}+K_2$$
$$=K_1F(\Phi|m)+K_2. \quad (9)$$

Evaluating the constants K_1, K_2 from the coordinates of the points P_1, P_2, P_3, and P_4, (Fig. 1), (9) is obtained as

$$W=u+jv=\frac{-F(\Phi|m)}{K(m)}-\frac{F\left(\sin^{-1}\sqrt{\frac{n}{m}}\middle|m\right)}{K(m)}+j\frac{K'(m)}{K(m)} \quad (10)$$

where U_0 and V_0 shown in Fig. 1(c) are given by

$$U_0=1+\frac{F\left(\sin^{-1}\sqrt{\frac{n}{m}}\middle|m\right)}{K(m)} \quad (11a)$$

$$V_0=\frac{K'(m)}{K(m)}. \quad (11b)$$

III. Characteristic Impedance

Half of the structure shown in Fig. 1(a) is transformed into parallel plates. The width of the parallel plates is 2 and V_0 is the separation between them. Therefore, the total capacitance per unit length of the line is twice that of the parallel plate capacitor in Fig. 1(c) and is of the form

$$C'=\frac{4\epsilon_0\epsilon_r}{V_0}. \quad (12a)$$

The characteristic impedance of the transmission line can be determined from the formula

$$Z_0=\frac{30\pi}{\sqrt{\epsilon_r}}\cdot V_0$$
$$=\frac{30\pi}{\sqrt{\epsilon_r}}\cdot\frac{K'(m)}{K(m)}. \quad (12b)$$

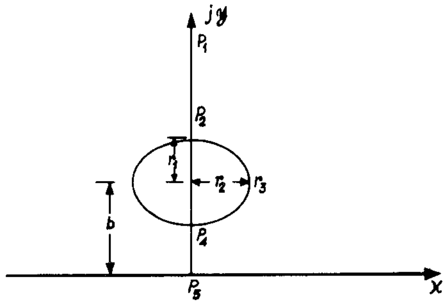

Fig. 2.

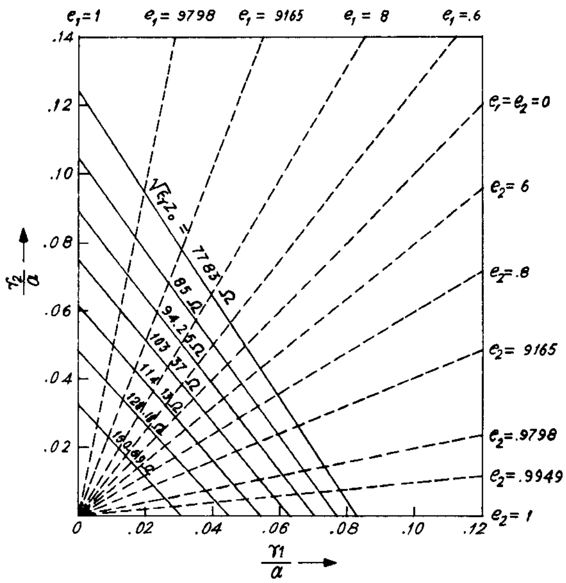

Fig. 3.

A. Elliptic Conductor

It is evident from (12b) that the characteristic impedance is dependent on the parameter m which is a function of the shape and dimensions of the center conductor of the transmission line.

For a known value of m and values of b/a equal to 0.1 and 0.4, values of n for λ varying from 0 to ∞ are determined from (5d). Then, r_1/a and r_2/a as well as the eccentricity of the ellipse can be computed. These results are presented as constant impedance contours in Fig. 2 and Fig. 3 in which r_1/a and r_2/a are coordinate axes. Impedance data are presented for only those values of the parameters for which the cross section is an ellipse. From a knowledge of r_1/a and r_2/a, the $e_1=$ constant, $e_2=$ constant contours can be easily determined. These contours are straight lines passing through the origin in the $r_1/a-r_2/a$ plane. For example, the intersection of a straight line, for which $e_1=e_2=0$, with the constant impedance contours of Figs 2, 3, and 4 gives the impedances of the circular conductor between ground planes. The cross section of the conductor under this condition is a circule for the values of the parameters of Figs. 2, 3, and 4. This straight line makes an angle 45° with the r_1/a axis. The

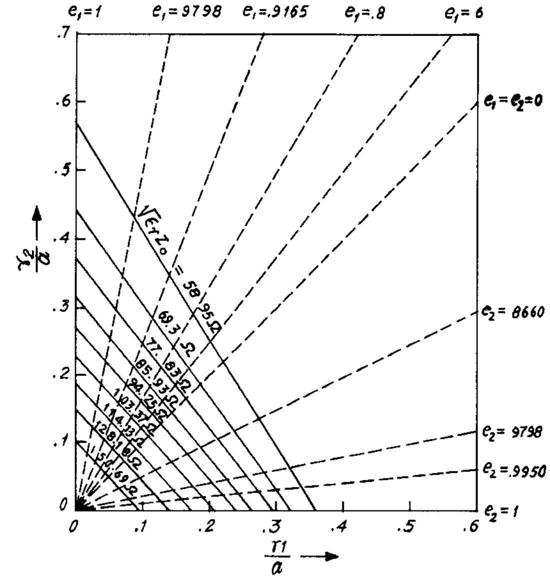

Fig. 4.

intersection of straight lines at angles greater than 45° (with respect to the r_1/a axis) and the constant impedance contours gives the impedances for elliptic conductor with major axis oriented parallel to ground planes. Inclinations less than 45° give ellipses with the major axis oriented perpendicular to the ground planes. The coordinate axes r_1/a and r_2/a represent cases for which the elliptic conductors degenerate to straight lines. Movement along any constant impedance contour from the r_2/a axis to the r_1/a axis corresponds to variation of λ from 0 to ∞.

B. Strip Conductor Parallel to Ground Planes

For $\lambda=0$, it is found from (5e) and (8a) that

$$\frac{r_1}{a}=0 \text{ and } e_1=1.$$

Thus, the ellipse degenerates to a straight line parallel to the ground planes. The corresponding transformation is given by

$$Z=C_1\Big[u-A_1\big\{\pi(n,u|m)-\sqrt{n}\,f(m,n,u)\big\}\Big]+B_1 \quad (13)$$

where the constants C_1, B_1, and A_1 are evaluated from (5b), (5c), and (5a) for $\lambda=0$. (13) is of the same form as that obtained by Rao and Das [13]. The corresponding impedances for this strip of zero thickness are obtained from the intersection of constant impedance contours with the r_2/a axis of Figs. 3 and 4.

C. Strip Conductor Perpendicular to Ground Planes

For $\lambda\to\infty$, it is clear from (5f) and 8(b) that

$$\frac{r_2}{a}=0 \qquad e_2=1 \quad \text{and}$$

$$\frac{r_1}{a}=\frac{1}{2\pi}\left\{\sin^{-1}\frac{m+\sqrt{n}}{\sqrt{m}\left(1+\sqrt{n}\right)}-\sin^{-1}\frac{-m+\sqrt{n}}{\sqrt{m}\left(1-\sqrt{n}\right)}\right\}. \quad (14)$$

The ellipse in this case degenerates to a straight line oriented in a direction perpendicular to ground planes. The impedances for this case and obtained from the intersection of the constant impedance contours with r_1/a axis of Figs. 3 and 4.

D. Round Conductor

For a circular conductor ($e_1 = e_2 = 0$) (8a), (8b), and (5g) give

$$\lambda = \frac{2\sqrt{m-n}\cdot Q(m,n)}{\sqrt{n}\,S(m,n)}. \tag{15}$$

Substituting (15) into (5a), (5b), (5c), and (5d) results in the conformal transformation for an offset circular conductor between ground planes.

IV. The Conductor Above A Single Ground Plane

A. Elliptic Conductor

When one of the ground planes is moved to infinity, the configuration of Fig. 1(a) reduces to the structure shown in Fig. 5. In this particular case, $a \to \infty$ and it is found from (5d) that $n \to m$.

Dividing (5e) and (5f) by (5d) the expressions for r_1/b and r_2/b are found. For $n \to m$ these expressions assume indeterminate $0/0$ form. Applying L'Hospital's rule and substituting the expression for elliptic integral of the third kind for $n = m$, it is found that

$$\frac{r_1}{b} = \frac{2\lambda E(m)\sqrt{m}}{\left[\pi\sqrt{1-m}+2\lambda E(m)\right]} \tag{16a}$$

$$\frac{r_2}{b} = \frac{2\sqrt{1-m}}{\left[\pi\sqrt{1-m}+2\lambda E(m)\right]}\cdot L(m) \tag{16b}$$

$$\lambda = \frac{\sqrt{1-m}\cdot L(m)}{\sqrt{m}\,E(m)}\cdot\left(\frac{r_1}{r_2}\right) \tag{16c}$$

where $E(m)$ is complete elliptic integral of the second kind and

$$L(m) = E(m)F\left(\sin^{-1}\frac{\sqrt{m}}{(2-m)}\middle| m\right) - K(m)E\left(\sin^{-1}\frac{\sqrt{m}}{(2-m)}\middle| m\right) + \frac{\sqrt{m}\,K(m)\sqrt{\left(m-2+\sqrt{m}\right)\left(m-2-\sqrt{m}\right)}}{(2-m)\sqrt{1-m}}. \tag{16d}$$

The corresponding Schwarz–Christeffel transformation ($n = m$) for transforming one half of the structure, shown in Fig. 5, into upper half plane of the Fig. 1(b) is given by

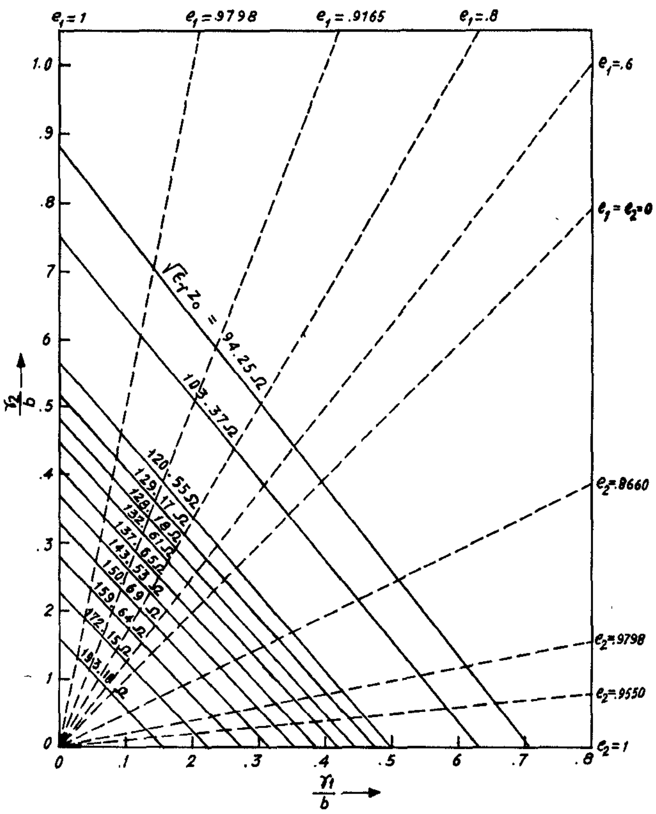

Fig. 5.

$$Z = \frac{-2b\sqrt{1-m}}{\left[\pi\sqrt{1-m}+2\lambda E(m)\right]}\cdot\left[E(m)F(\Phi|m) - K(m)E(\Phi|m) - \sqrt{m}\,K(m)\frac{\sqrt{1-t^2}}{\sqrt{1-mt^2}}\left\{1-\sqrt{mt}\right\} + j\lambda E(m)\left\{\frac{1+\sqrt{m}\,t-\sqrt{1-mt^2}}{1+\sqrt{m}\,t}\right\}\right] + jb - \frac{2jb\lambda E(m)\left(1-\sqrt{1-m}\right)}{\left[\pi\sqrt{1-m}+2\lambda E(m)\right]} \tag{17}$$

where $t = \sin\Phi = \mathrm{sn}\,u$ and $E(\Phi|m)$ is an incomplete elliptic integral of the second kind.

The transformation which maps the upper half plane of Fig. 1(b) into a rectangle of Fig. 1(c) can be obtained from (10) with the substitution $n = m$.

Using the method as discussed in Section III, the characteristic impedance of the structure of Fig. 5 is calculated from (16a) and (16b). Results are presented as constant impedance contours in Fig. 4.

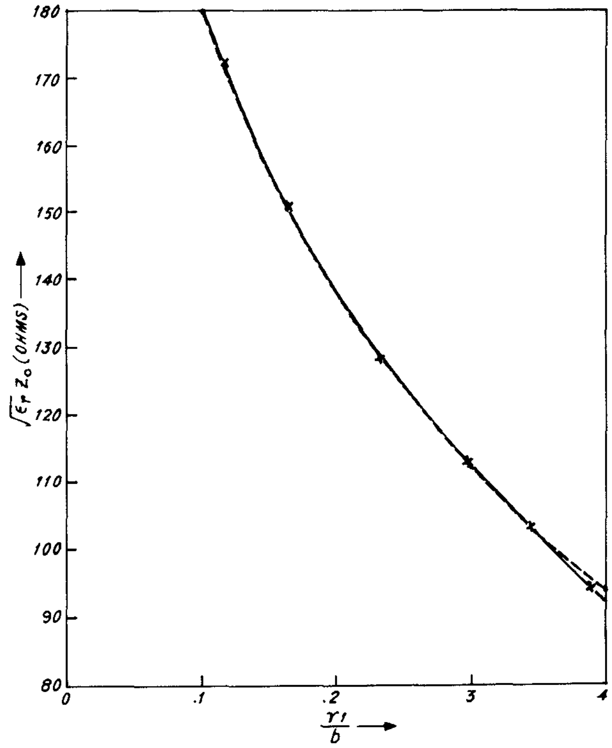

Fig. 6. ✕✕✕ is the present method. ——— is by the method in literature.

B. Strip Conductor

For $\lambda = 0$ a structure consisting of a planar strip above a ground plane is obtained and the expression (17) reduces to that obtained by Joshi, Rao, and Das [14].

For $\lambda \to \infty$

$$\frac{r_2}{b} = 0 \text{ and } e_2 = 1 \tag{18a}$$

$$\frac{r_1}{b} = \sqrt{m}\,. \tag{18b}$$

These expressions can be used for the analysis of a vertical strip above an infinite ground plane. The impedance for this structure is obtained from the intersection of constant impedance line with the r_1/b axis of Fig. 4.

C. Round Conductor

For the case of a circular conductor above a ground plane

$$e_1 = e_2 = 0 \text{ and}$$

$$\lambda = \frac{\sqrt{1-m}}{\sqrt{m}} \cdot \frac{L(m)}{E(m)}. \tag{19}$$

By using the value of the λ from (19) in (17), the conformal transformation in the case of a circular conductor placed above a single ground plane is obtained.

The comparison of the impedance data for a circle above a ground plane with those evaluated using the method of analysis suggested by Decreton [11] is presented in Fig. 6.

V. Summary

The analysis presented in this paper is general and embraces the cases of planar, elliptic, and circular conductors arbitrarily located between ground planes or above a single ground plane. Depending upon the location of the conductor between ground planes, there is a maximum value of m for which the boundary of the conductor is an ellipse. Impedance data for a circle above a ground plane, evaluated by the present method show an excellent agreement with those computed from Decreton [11]. In the special case of a planar strip conductor, the present formulation leads to expressions for the conformal transformation which are identical with those obtained previously [13], [14].

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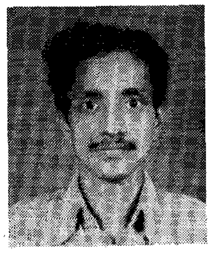

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Calibration of Multiport Reflectometers by Means of Four Open/Short Circuits

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***Abstract*—This paper presents a simple method for calibrating any practical multiport reflectometer by means of four reflection standards with known complex reflection coefficients. It is shown that these four standards can be such that their reflection coefficient modulus = 1. Computer simulation proves that no singularity appears for both ideal and nonideal five- and six-port reflectometer in a wide range of phase distribution of reflection coefficients. A group of calibration results for a practical simple six-port is listed to show this calibration procedure; by the use of these calibrated network parameters, some measurement results are presented and compared with the values obtained at the National Bureau of Standard, U.S.A.**

Both computer simulation and experimental results show that the numerical singularities which may be encountered in multiport calibration procedures are not an intrinsic properties of multiport but from related mathematical treatment.

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I. Introduction

IT IS WELL KNOWN that the key problem for a network analyzer is its calibration. The existing self-calibration procedures for the six-port reflectometer [1], [3] can provide accurate results but are complex and cannot be directly used to calibrate the five-port. Another way to calibrate a network analyzer is via some reflection standards, which would be very useful for microwave engineering application. Woods [4] has discussed this problem in detail and concludes that seven standards are needed, of which at most five may have $|\Gamma| = 1$, to avoid numerical

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